Linear Algebra Homework Set 1

Due: March 21, 2008

1. What conditions must the constants, the *b*'s, satisfy so that the linear equation system has a solution? You can apply Gauss' method and see what happens to the right side.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= b_1 \\ 2x_1 + 5x_2 + 3x_3 &= b_2 \\ x_1 \qquad\quad + 8x_3 &= b_3 \end{aligned}$$

2. (a) Solve the system of equations.

$$\begin{aligned} ax + y &= a^2 \\ x + ay &= 1 \end{aligned}$$

For what values of *a* does the system fail to have solutions, and for what values of *a* are there infinitely many solutions?

(b) Answer the above question for the system.

$$\begin{aligned} ax + y &= a^3 \\ x + ay &= 1 \end{aligned}$$

3. Is the given vector in the set generated by the given set?

$$(a) \quad \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\}$$

$$(b) \quad \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \left\{ \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \right\}$$

4. Three truck drivers went into a roadside cafe. One truck driver purchased four sandwiches, a cup of coffee, and ten doughnuts for $8.45. Another driver purchased three sandwiches, a cup of coffee, and seven doughnuts for $6.30. What did the third truck driver pay for a sandwich, a cup of coffee, and a doughnut?

5. Use the computer to solve the following problem

$$i_0 - i_1 - i_2 = 0$$
$$i_1 - i_3 - i_5 = 0$$
$$i_2 - i_4 + i_5 = 0$$
$$i_3 + i_4 - i_6 = 0$$
$$5i_1 + 10i_3 = 10$$
$$2i_2 + 4i_4 = 10$$
$$5i_1 - 2i_2 + 50i_5 = 0$$

You shall seek help from Mathematica, Maple, or write your own C program. List your code as you use it to find your answer.